GCSE MATHEMATICS

S21-C300

With Calculator Assessment Resource O

Higher Tier

Formula list

Area and volume formulae

Where r is the radius of the sphere or cone, l is the slant height of a cone and h is the perpendicular height of a cone:

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Kinematics formulae

Where a is constant acceleration, u is initial velocity, v is final velocity, s is displacement from the position when $t = 0$ and t is time taken:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

1. The table below gives information from the Highway Code on stopping distances for cars.

Speed	**Stopping distance in metres = Thinking distance + Braking distance** (Thinking distance is given first, followed by Braking distance)	
20 mph	6 m	6 m
30 mph	9 m	14 m
40 mph	12 m	24 m
50 mph	15 m	38 m

Remember 50 mph is 80 km/h.

The stopping distances given in the Highway Code assume good driving conditions and alert drivers.

When a driver is tired and the road is wet, the thinking distance increases by 30% and the braking distance increases by 20%.

A tired driver travels at 64 km/h in wet driving conditions.

Calculate their stopping distance in metres. [4]

80 km/h = 50 mph

1 km/h = $\frac{5}{8}$ mph

∴ 64 km/h = 40 mph

Thinking distance = 12 × 1.3 = 15.6

Braking distance = 24 × 1.2 = 28.8

15.6 + 28.8 = 44.4

Stopping distance = 44.4 m

2. *(a)* In Queenbridge, the mean daily snowfall for a week was 1·6 cm.
If there had been <u>1 cm</u> more snowfall on each day, what would the mean daily snowfall have been? [1]

2 · 6 cm

$\left(\frac{x}{7} = 1 \cdot 6 \rightarrow x = 11 \cdot 2 \qquad \frac{11 \cdot 2 + 7}{7} = 2 \cdot 6 \text{ cm} \right)$

(b) In Sansburg, the snowfall for each of the first 10 days in January was measured.
The results are summarised in the table below.

Daily snowfall, s in cm	Number of days (f)	x	fx
$1{\cdot}5 \leqslant s < 2{\cdot}5$	4	2	8
$2{\cdot}5 \leqslant s < 3{\cdot}5$	2	3	6
$3{\cdot}5 \leqslant s < 4{\cdot}5$	1	4	4
$4{\cdot}5 \leqslant s < 5{\cdot}5$	0	5	0
$5{\cdot}5 \leqslant s < 6{\cdot}5$	3	6	18
	= 10		= 36

Calculate an estimate for the mean daily snowfall for these 10 days. [4]

$\tilde{x} = \frac{36}{10} = 3 \cdot 6$

∴ mean daily snowfall is 3·6 cm

(c) During the first 5 days of February, the mean snowfall in Awezell was 4·7 cm.
On 6th February the snowfall was 23·9 cm.

Calculate the mean snowfall for the first 6 days of February. [3]

$\frac{x}{5} = 4 \cdot 7 \rightarrow x = 23 \cdot 5$

$\frac{23 \cdot 5 + 23 \cdot 9}{6} = 7 \cdot 9$

7 · 9 cm

3. *(a)* In 2015, the average price of coal sold by a mine in the USA was \$31.83 per ton.
This coal was then delivered to power stations.
The power stations paid \$42.58 per ton for this coal.
The difference in price was the delivery cost.

What percentage of the original price of the coal was this delivery cost?
Give your answer correct to 3 significant figures. [3]

MINE \$ 31.88 / ton.

STATION \$ 42.58 / ton

DELIVERY = 42.58 − 31.83 = \$10.75 / ton

$\frac{10.75}{31.83} \times 100 = 33.773169 = 33.8\%$ (3sf)

(b) In the USA and the UK the word 'ton' means different amounts:
- a UK ton = 1016 kg,
- a USA ton = 907 kg,
- a tonne (called a metric ton in the USA) = 1000 kg.

What is the difference between a UK ton and a USA ton?
Give your answer in tonnes. [2]

1016 − 907 = 109 kg difference

109kg ⟶ 0.109 tonnes

(c) A preformed piece of coal weighs 100 g **correct to the nearest 5 g**.
A bag contains 30 pieces of preformed coal.

Complete the sticker to attach to this bag of coal. [4]

This bag of coal weighs **at least**

2.85 **kg**

a piece of coal is at least 95 g

95g = 0.095 kg

0.095 × 30 = 2.85 kg

4. Candice has been given a bracelet.

The dimensions of the bracelet are given below.

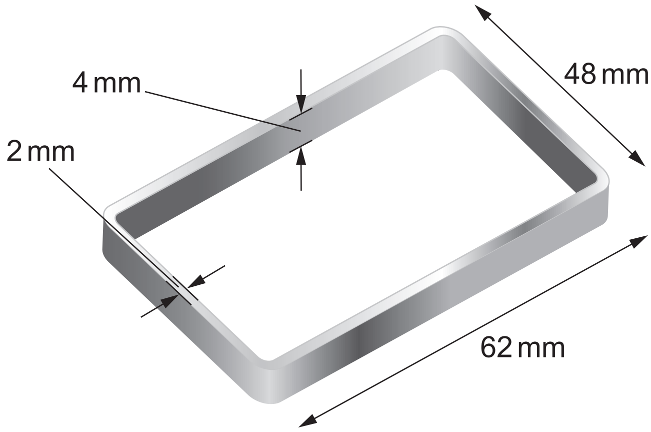


Diagram not drawn to scale

Candice knows it is made entirely from one metal. She is not sure if it is copper, silver or gold. Her bracelet has a mass of approximately 18 g.

Metal	Density (g/cm^3)
Copper	8·96
Silver	10·49
Gold	19·32

Convince Candice that her bracelet is made from silver.
You must show all your working.

0.992 * 0.768 [5]

density = $\frac{\text{mass}}{\text{volume}}$

density x volume = mass

densty x voume = 18g

Volume of bracelet

$(48 \times 62 \times 4) - (44 \times 58 \times 4)$

$= 1696 \text{ mm}^3 = 1.696 \text{ cm}^3$

$18 \div 1.696 = 10.61 \approx 10.49$

10.61 g/cm³ is closest to 10.49 thus the bracelet is made out of silver.

5. Solve the following simultaneous equations.

$$y = 3x^2 + 4x - 7$$
$$y = 2x + 5$$

Use an algebraic method and give your answers correct to 2 decimal places. [6]

$2x + 5 = 3x^2 + 4x - 7$

$0 = 3x^2 + 2x - 12$

quadratic formula $= \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

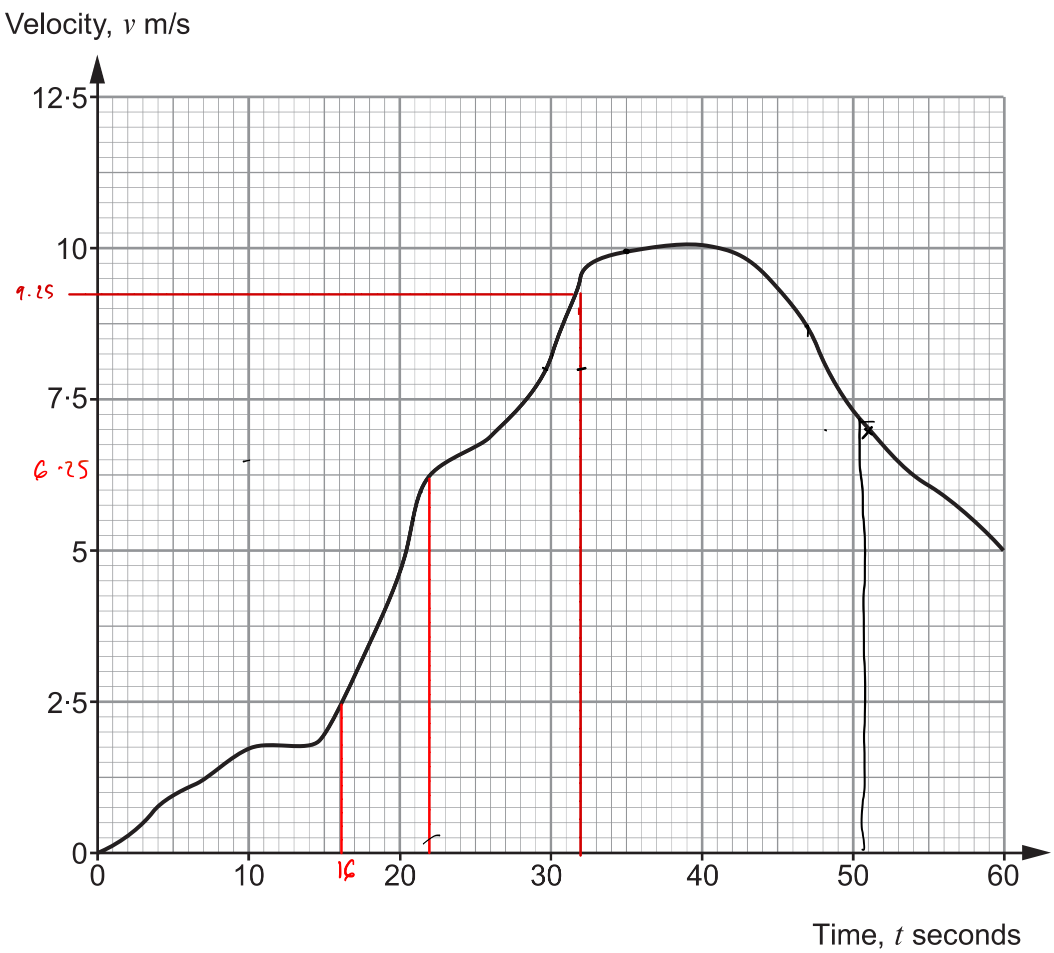
$\dfrac{-2 + \sqrt{4 + 144}}{6} = \dfrac{-2 + 2\sqrt{37}}{6}$ | $\dfrac{-2 - \sqrt{4 + 144}}{6} = \dfrac{-2 - 2\sqrt{37}}{6}$

$x = \dfrac{-1 + \sqrt{37}}{3}$ OR $x = \dfrac{-1 - \sqrt{37}}{3}$

$= 1.69425..$ $\quad = -2.3609..$

$= 1.69$ $\quad = -2.36$

6. The velocity-time graph below shows the first 60 seconds of a car's journey.



(a) After how many seconds was the velocity of the car 9 **km/h**? [4]

$9\,km/h \rightarrow \frac{9}{3.6} = 2.5 \left(= \frac{9000\,m}{3600\,s}\right)$

$\therefore 9\,km/h = 2.5\,m/s$

After 16 seconds

(b) Harriet argues that the acceleration at $t = 22$ represents the typical acceleration of the car during the first 32 seconds of this period.

Explain why Harriet's argument is correct. [1]

acceleration at $t = 22 = \frac{6 \cdot 25}{22} = 0 \cdot 284...$ $\approx 0 \cdot 3$

acceleration for 32 seconds $= \frac{9 \cdot 25}{32} = 0 \cdot 289...$ $\approx 0 \cdot 3$

(c) Over the same 60 seconds, the velocity, v m/s, at time, t seconds, of another car is given by the following equation.

$$v = 7 + \frac{t^2}{1000}$$

Find two times for which the difference in the two cars' velocities was 2·5 m/s.
Give these times correct to the nearest second.
You must show all your working. [4]

$7 + \frac{t^2}{1000}$ TRIAL + IMPROVEMENT

$\rightarrow 7 + \frac{20 \cdot 5^2}{1000} = 7 \cdot 42025$

car 1 @ 20·5 s = 5 $\rightarrow$ 7·42 − 5 = 2·42 ≈ 2·5

∴ 20·5 → 21 seconds

$7 + \frac{51^2}{1000} = 9 \cdot 601$

car 1 at 51 s = 7

9·6 − 7 = 2·6

∴ 21 seconds to the nearest second and 51 seconds to the nearest second